Monomial crystals and promotion operators

Description of the $\mathcal{U}_q(\hat{sl}_4)$-crystal $\mathcal{M}(Y_{1,1}\, Y_{1,-1}\, Y_{0,2}^{-1}\, Y_{0,0}^{-1})$

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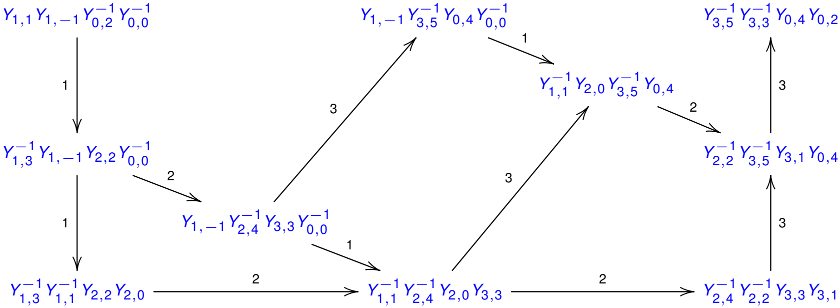
We describe the crystal $\mathcal{M}_{NC} = \mathcal{M}(Y_{1,1} Y_{1,-1} Y_{0,2}^{-1} Y_{0,0}^{-1})$ of type A_3 used in Section 2.4 of the thesis. Let $\phi : \mathcal{M}_{NC} \to \mathcal{M}_{NC}$ be the map defined by

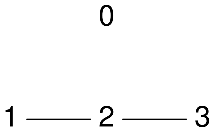
$$\phi(\prod Y_{i,n}^{u_{i,n}}) = \prod Y_{i+1,n+1}^{u_{i,n}}.$$

We give

- the sub-I_j-crystals $\mathcal{M}_j = \mathcal{M}_{I_j}(\phi^j(Y_{1,1} Y_{1,-1} Y_{0,2}^{-1} Y_{0,0}^{-1}))$ for $j = 0, \ldots, 4$,
- the corresponding Dynkin diagram,
- the $\mathcal{U}_q^{v,j}(sl_4^{tor})$-module associated to $\mathcal{M}_j$ whose q-character is
$$\Xi^j(\sum_{m \in \mathcal{M}_j} m).$$

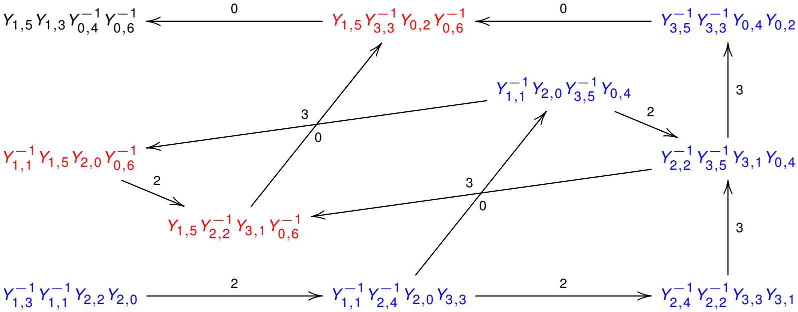
The action of the promotion operator ϕ on $\mathcal{M}_{NC}$ can be viewed as a screwing.

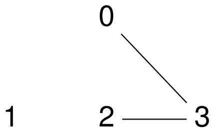




Associated $\mathcal{U}_q^{v,0}(sl_4^{tor})$-module :

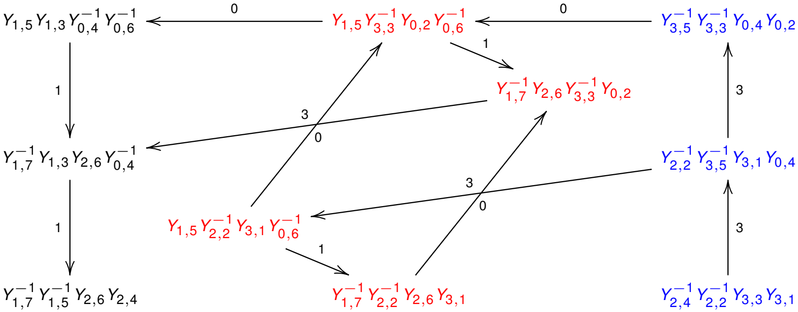
$$V(\Xi^0(Y_{1,1} Y_{1,-1} Y_{0,2}^{-1} Y_{0,0}^{-1}))$$

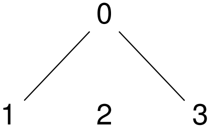




Associated $\mathcal{U}_q^{v,1}(sl_4^{tor})$-module :

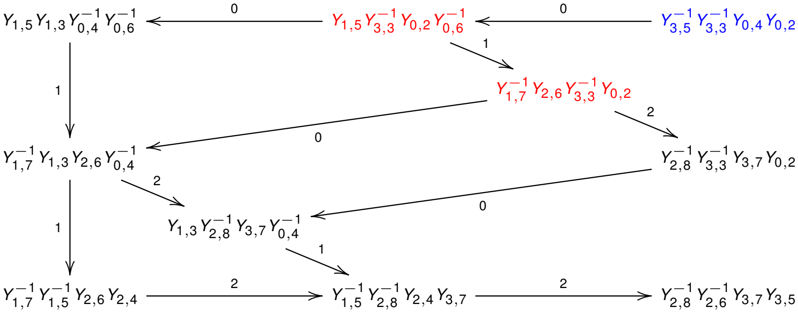
$$V(\Xi^1(Y_{1,3}^{-1}Y_{1,1}^{-1}Y_{2,2}Y_{2,0}))$$

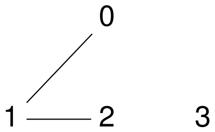




Associated $\mathcal{U}_q^{v,2}(sl_4^{tor})$-module :

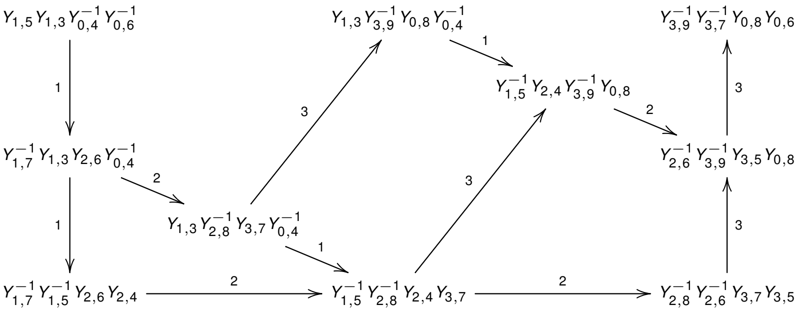
$$V(\Xi^2(Y_{2,4}^{-1}Y_{2,2}^{-1}Y_{3,3}Y_{3,1}))$$

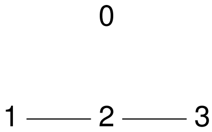




Associated $\mathcal{U}_q^{v,3}(sl_4^{tor})$-module :

$$V(\Xi^3(Y_{3,5}^{-1}Y_{3,3}^{-1}Y_{0,4}Y_{0,2}))$$



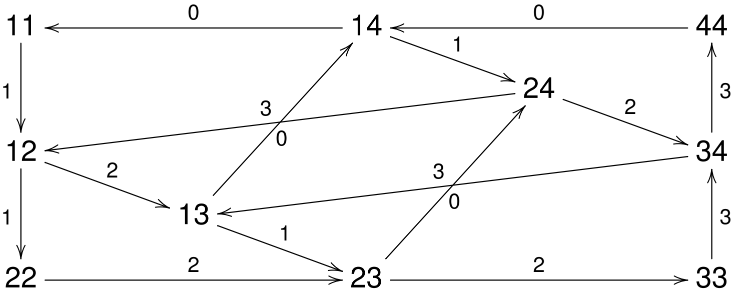


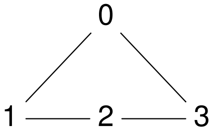
Associated $\mathcal{U}_q^{v,0}(sl_4^{tor})$-module :

$$V(\Xi^0(Y_{1,5}Y_{1,3}Y_{0,4}^{-1}Y_{0,6}^{-1}))$$

Link with the P_{cl}-crystal $\mathcal{B}_0(2\Lambda_1)_{\mathrm{aff}}$

Let us give the P_{cl}-crystal $\mathcal{B}_0(2\Lambda_1)_{\mathrm{aff}}$ of type $A_3^{(1)}$ obtained by promotion operator pr. In the crystal, i j $=$ $\boxed{i \mid j}$ for all i, j.





Crystal graph of the $\mathcal{U}_q(\hat{sl}_4)'$-module $W(2\varpi_1)$